The Formation of Fragments at Corotation in Isothermal Protoplanetary Disks

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ABSTRACT

Numerical hydrodynamics simulations have established that disks which are evolved under the condition of local isothermality will fragment into small dense clumps due to gravitational instabilities when the Toomre stability parameter Q is sufficiently low. Because fragmentation through disk instability has been suggested as a gas giant planet formation mechanism, it is important to understand the physics underlying this process as thoroughly as possible. In this paper, we offer analytic arguments for why, at low Q, fragments are most likely to form first at the corotation radii of growing spiral modes, and we support these arguments with results from 3D hydrodynamics simulations.

Subject headings: accretion, accretion disks — hydrodynamics — instabilities — planetary systems: formation — planetary systems: protoplanetary disks

1. Introduction

The idea that gas giant planets can form rapidly through gravitational instabilities (GIs) in protoplanetary disks (Boss 1997, 1998), usually called the "disk instability" theory, has now been subjected to intensive study with numerical hydrodynamics techniques (Pickett et al. 1998, 2000, 2003; Nelson et al. 1998, 2000; Nelson 2006; Boss 2000, 2001, 2002, 2003, 2005, 2007; Gammie 2001; Mayer et al. 2002, 2004, 2007; Johnson & Gammie 2003; Rice et al. 2003, 2005; Mejía et al. 2005; Cai et al. 2006; Boley et al. 2006, 2007). Although all investigators agree that massive, cold protoplanetary disks fragment into dense clumps under conditions of rapid cooling or local isothermality, there is not universal agreement that these fragments are sufficiently long-lived to be considered bound gas giant protoplanets or that fragmentation will occur under realistic disk conditions (Durisen et al. 2003, 2007; Boss 2004, 2005; Rafikov 2005, 2006; Pickett & Durisen 2007a). This important question may ultimately be decided through simulations alone, but we will gain greater confidence in the numerical results if we understand the physics of fragmentation and clump longevity through analytic arguments, even if only approximate.

As a first step in this direction, we consider the special case of a disk evolved under a "locally isothermal" assumption, i.e., the disk is assumed to maintain its initial temperature at all positions (Boss 1998; Pickett et al. 1998; Nelson et al. 1998). For brevity, most GI researchers refer to this as an "isothermal" disk, even though the temperature of the disk is not the same at all positions. Local, thin-disk simulations with a variety of Qs by Johnson & Gammie (2003) suggest that fragmentation of isothermal disks occurs for $Q <$ about 1.4, and the set of all global 3D hydrodynamics simulations for isothermal disks referenced in the preceding paragraph generally supports the occurrence of fragmentation for such low Qs. The same set of simulations, taken together, also generally confirms that disks are subject to growth of nonaxisymmetric GIs for $Q <$ about 1.7 or so and are stable

for higher Qs. In simulated disks that exhibit GIs, the instabilities are initiated by the growth from noise of discrete spiral modes (Pickett et al. 1998, 2000, 2003). Our own latest isothermal disk simulations, which will be reported in detail elsewhere (Pickett & Durisen 2007b), show three types of behavior: 1) at low Q, *prompt* fragmentation of spiral GI modes as they first become nonlinear, 2) at intermediate Q, *delayed* fragmentation in the nonlinear regime, probably due to nonlinear interactions of multiple spiral modes, and 3) at the highest unstable Qs, nonlinear spirals which do not fragment.

In this paper, we explore the idea that prompt fragmentation in isothermal disks tends to occur near the radius at which a discrete growing spiral mode corotates with the disk gas (the "corotation radius" or CR). In Section 2, we give simple analytic arguments that compression by spiral shocks must not be too large if the shock-compressed gas is going to be able to fragment and that the smallest compression in a spiral mode should be found near its CR. In Section 3, we describe representative results from a simulation in Pickett & Durisen (2007b) where prompt fragmentation does in fact occur in the vicinity of the CR for a discrete fast-growing mode at low Q. The simulation allows us to estimate some of the uncertain factors in the analysis of Section 2 and verify that the analytic arguments do apply. Section 4 summarizes our main conclusions.

2. Compression-Induced Stability Against Fragmentation

Consider cylindrical coordinates (r,ϕ,z) with the rotation axis of the disk along the z-axis, the disk midplane at $z = 0$, and the sense of rotation in the positive ϕ-direction. In the absence of spiral structures and shocks, a low-Q disk has a half-thickness $H \ll r$ in the z-direction defined such that $\Sigma_1 = 2\rho_1 H$, where Σ_1 is the surface density and ρ_1 is the midplane density. We shall assume that the initially fastest-growing discrete spiral mode develops more rapidly than any other instability which may give rise to fragments. A spiral

shock at r associated with the mode then sweeps material into a sheet of vertical height H with thickness L normal to the spiral shock front. Because the gas responds isothermally to the shock, there is no vertical jump behind the shock, and the vertical height of the disk remains essentially the same on both sides of the shock (Boley & Durisen 2006).

We will analyze the gravitational stability of the swept-up gas by assuming that it is well approximated by a thin, plane-parallel sheet. This requires $L \ll r$ and

$$L < 2\alpha_1 H, \tag{1}$$

where α_1 is a constant of order unity which accounts for uncertainty about how much smaller L must be than $2H$ for our subsequent analysis to be at least approximately valid. Throughout, we will introduce similar factors, each of which is presumably of order unity and designated by an α_j, where j is an index. They account for uncertainties associated with our assumptions.

If the density ρ_2 in the swept-up sheet can be considered roughly constant over scales of order L, which seems reasonable, then potential energy per unit area of the gas sheet is

$$E_G = -\pi G \rho_2{}^2 L^3/6, \tag{2}$$

where G is the gravitational constant. There are several effects that can counter the tendency of self-gravity to cause fragmentation, including thermal pressure, tidal gravitational stresses, and shearing motions. The Virial Theorem suggests that an infinite, isothermal, plane-parallel, ideal gas slab will be stable against fragmentation due to its thermal energy alone if $|E_G| < 2E_T$, where E_T is the total thermal kinetic energy of the slab per unit area. This gives

$$\pi G \rho_2 L^2 < 18\alpha_2 c_{\mathrm{i}}{}^2, \tag{3}$$

where c_{i} is the isothermal sound speed and α_2 accounts for the uncertainty due to deviations

from a thin, uniform slab geometry. Stability condition (3) simplifies to

$$\Sigma_{\text{slab}}{}^2/\rho_2 < 18\alpha_2 c_{\text{i}}{}^2/\pi G, \tag{4}$$

where $\Sigma_{\text{slab}} = \rho_2 L$ is the surface mass density of the swept up slab of gas in one of the spiral arms. If the spiral mode has m equally-spaced arms that together have swept up a fraction f of the material in each annulus of the disk at the time of fragmentation, then

$$\Sigma_{\text{slab}} = 2\pi r f \rho_1/m. \tag{5}$$

From equation (4), we see that, for a fixed value of Σ_{slab}, fragmentation is inhibited by large values of ρ_2. In other words, strong compression by spiral shocks *suppresses* fragmentation. This somewhat counterintuitive result is peculiar to isothermal slabs. Compression by the spiral shock is least near the radius at which the spiral pattern corotates with the disk orbital motion. At significant distances from CR, shock compression will be strong and will suppress fragmentation. A major aim of this section is to estimate the minimum distance away from the CR at which this suppression is effective.

We first examine criterion (1) by noting that

$$L = 2\pi r f \rho_1/m\rho_2. \tag{6}$$

If the shock is strong and isothermal, then

$$\rho_2/\rho_1 = v_{\text{sh}}{}^2(\sin\psi)^2/c_{\text{i}}{}^2, \tag{7}$$

where v_{sh} is the ϕ-direction speed of the spiral shock front relative to the pre-shock material and ψ is the angle to the normal of the spiral shock front made by the fluid streamlines coming into the shock. At a distance δr from the CR of the spiral mode,

$$v_{\text{sh}} = -3\alpha_3\Omega\delta r/2, \tag{8}$$

where $\delta r/r \ll 1$ and Ω is the disk's undisturbed angular frequency of rotation. The factor $\alpha_3 = 1$ for a disk in Keplerian rotation but may differ from unity for disks with significant self-gravity or pressure support. We define α_4 such that

$$H = \alpha_4 c_i/\Omega. \tag{9}$$

Using (6) through (9), we find that the thin-sheet condition (1) is satisfied if

$$\left(\frac{\delta r}{H}\right)^2 > \frac{4\pi r f}{9\alpha_1 \alpha_3{}^2 \alpha_4{}^2 (\sin\psi)^2 mH}. \tag{10}$$

Now let us consider criterion (3) for compression-induced suppression of fragmentation. For disk gas that is strictly isothermal, the Toomre (1964) stability parameter Q_i is

$$Q_i = c_i \kappa/\pi G\Sigma_1, \tag{11}$$

where the epicyclic frequency κ is $[d(r^2\Omega)^2/r^3 dr]^{1/2}$ (Binney & Tremaine 1987). We set

$$\kappa = \alpha_5 \Omega, \tag{12}$$

so that $\alpha_5 = 1$ for a Keplerian disk. Using (4), (5), (7), (8), (9), (11), and (12) plus the definition of Σ_1, we get that condition (3) for suppression of fragmentation is satisfied if

$$\left(\frac{\delta r}{r}\right)^2 > \frac{4\pi^2 \alpha_5 f^2}{81\alpha_2 \alpha_3{}^2 \alpha_4 (\sin\psi)^2 m^2 Q_i}. \tag{13}$$

Together inequalities (11) and (13) show that, for sufficiently large values δr, both conditions (1) and (3) are met. In other words, away from the CR of a growing spiral mode with m arms, shock compression due to the mode creates a thin sheet of gas which is stable against fragmentation. If prompt fragmentation is to occur, it must do so in the vicinity of the CR. According to (11) and (13), as m decreases, the region around the CR where the post-shock slab is not stabilized increases. Whether prompt fragmentation will occur thus depends on what mode with what corotation radius is likely to reach nonlinear amplitude first.

So far, there is no rigorous analytic theory which predicts the number of arms for the fastest growing nonaxisymmetric mode of a gravitationally unstable disk. However, a WKB analysis for axisymmetric modes of thin disks with $Q < 1$ (Toomre 1964; Binney & Tremaine 1987) yields a wavelength for the fastest growing ring-like mode of

$$\lambda_{\mathrm{f}} \approx 2.2\pi^2 G\Sigma_1/\kappa^2. \tag{14}$$

Comparisons between simulations and equation (14) (Durisen et al. 2003) suggest that, for disks unstable to nonaxisymmetric modes, the fastest growing nonaxisymmetric mode has a number of arms given by

$$m_{\mathrm{f}} = \alpha_6 \pi r/\lambda_{\mathrm{f}} = \alpha_4\alpha_5\alpha_6 Q_{\mathrm{i}}r/2.2H, \tag{15}$$

where $\alpha_6 \sim 1$. Disks do not break up directly into fragments of this size. Instead, the simulations show that the mode grows to nonlinear amplitude as a spiral wave. In the case of prompt fragmentation, fragments appear first as a smaller-scale instability in the dense post-shock region associated with the nonlinear wave. If we set $m = m_{\mathrm{f}}$ in (10) and (13), the thin-sheet criterion (1) becomes

$$\left(\frac{\delta r}{H}\right)^2 > \frac{8.8\pi f}{9\alpha_1\alpha_3{}^2\alpha_4{}^3\alpha_5\alpha_6(\sin\psi)^2 Q_{\mathrm{i}}} = \frac{3.1\zeta}{\alpha_1 Q_{\mathrm{i}}} , \tag{16}$$

and criterion (3) for compression-induced stability against fragmentation becomes

$$\left(\frac{\delta r}{H}\right)^2 > \frac{0.24\pi^2 f^2}{\alpha_2\alpha_3{}^2\alpha_4{}^3\alpha_5\alpha_6{}^2(\sin\psi)^2 Q_{\mathrm{i}}^3} = \frac{2.4f\zeta}{\alpha_2\alpha_6 Q_{\mathrm{i}}^3} , \tag{17}$$

where

$$\zeta = f/\alpha_3{}^2\alpha_4{}^3\alpha_5\alpha_6(\sin\psi)^2. \tag{18}$$

The similar numerical quantities on the right hand sides of conditions (16) and (17) suggest that, whenever the spiral shock compresses gas into a thin slab, it will be stable against fragmentation. However, this also means that the thinness and strong shock

assumptions required for use of relations (3) and (7) break down together simultaneously near the CR. We cannot invert the inequalities to obtain conditions for instability because there is at least one other stabilizing effect, namely, shear. So, strictly speaking, our analysis does not tell us whether or when prompt fragmentation *does* occur, but it implies that, *if* prompt fragmentation occurs, then it must happen in the vicinity of the CR for conditions under which the inequalities are reversed. We can see from the Q_i-dependence of (17) that this is more likely to happen for low Q_i.

3. Comparison with Simulations

3.1. Methods and Initial Conditions

Adopting the same basic star/disk model and 3D hydrodynamics code used by Pickett et al. (1998), we have computed a series of locally isothermal simulations of disks with different, but constant, values of Q. The r, z-resolution is the same as in Pickett et al. (1998), but the ϕ-direction resolution is increased from 128 to 512 azimuthal cells in 2π radians. This seems sufficient to resolve fragmentation in modes of moderate m and to satisfy the Truelove et al. (1997) and Nelson (2006) criteria for avoiding purely numerical fragmentation prior to the onset of fragmentation. So far, the series of simulations includes $Q = 1$, 1.15, 1.25, 1.35, 1.5, 1.6, and 1.7. For reasons explained in Section 3.2 below, Q here, without the subscript "i", is computed using (11) but replacing the isothermal sound speed by the adiabatic sound speed for an ideal gas with ratio of specific heats $\gamma = 5/3$. Consequently,

$$Q = (5/3)^{1/2}Q_i = 1.29Q_i \qquad (19)$$

The star/disk model is the same massive "stubby" disk used in Pickett et al. (1998, 2000), with disk-to-star mass and radius ratios of about $M_d/M_s = 0.25$ and $R_d/R_s =$

7.1, respectively. In this case, however, in order to avoid the severe cost of the Courant time-step limitation near the rotation axis, the disk is detached from the star using the localized cooling method described in Pickett et al. (2003), and the stellar mass distribution and gravitational potential are frozen.

3.2. Prompt Fragmentation at Corotation

In the full Q-survey, which will be reported in greater detail elsewhere (Pickett & Durisen 2007b), the initial disk is given a small amplitude, random, cell-to-cell density perturbation. This allows fast growing modes to organize themselves and grow in a few dynamic times. The three behaviors described in the Introduction are found over the following ranges: 1) $1.0 \leq Q \leq 1.25$: Prompt Fragmentation. Fragments appear as soon as growing modes reach nonlinear amplitude. 2) $1.35 \leq Q \leq 1.6$: Delayed Fragmentation. Discrete modes do not fragment as soon as they reach nonlinear amplitude; instead, the appearance of dense fragments is delayed by several to many pattern rotations and appears to be associated with nonlinear interactions of different patterns or arms. 3) $Q = 1.7$: Nonfragmenting Spiral Arms. The disk is unstable and develops strong spiral arm structure, but no fragmentation occurs over the duration of a long simulation. We have not yet tested the upper bound of GI stability for this star/disk model, but we do know from Pickett et al. (1998) that a $Q = 2.0$ disk is stable against growth of nonaxisymmetric structure, so the Q-limit for onset of GIs is somewhere between 1.7 and 2.0. Clearly, the Q below which fragmentation occurs for these isothermally evolved disks is between 1.6 and 1.7. We compare this fragmentation limit with that found by Johnson & Gammie (2003) in Section 3.2 below.

For the low Qs of interest here, where prompt fragmentation occurs, many modes grow rapidly at once, and it is difficult to discern which mode or pattern period is associated

with a given fragment or set of fragments. So, for this paper, we apply Fourier analysis techniques (Pickett et al. 1998, 2000) to the $Q = 1.25$ case and determine, as best as we can, that the fastest-growing mode is a particular $m = 4$ mode. We then run an additional simulation with a pure, low-amplitude $\delta\rho/\rho \sim \cos4\phi$ density perturbation over the radial range where the fastest growing $m = 4$ pattern was detected in the random perturbation simulation. As a cautionary note, we point out that it is difficult to be sure we have truly isolated the *most* unstable mode, because many modes grow with similarly fast growth rates. A midplane greyscale of the disk close to the moment of prompt fragmentation in the run with the pure $\cos4\phi$ hit is shown in Fig. 1, where it is apparent that the fragments do indeed appear near the CR of the growing mode.

We can use the simulation to estimate some, but not all of the various parameters that enter into our analysis in Section 2. The rotational shear of this massive disk is somewhat non-Keplerian, so that $\alpha_3 \approx 0.9$ and $\alpha_5 \approx 0.9$. Using Σ_1 and the midplane values of ρ_1, c_i, and Ω near the CR in (9), we find that $\alpha_4 \approx 0.6$. Use of these α_js and the known values of Q_i, H, the r of CR, and $m_f = 4$ in (15) gives $\alpha_6 \approx 0.7$. It is difficult to measure ψ in our Eulerian code, but it is relatively easy to determine the pitch angle of the spirals to be about 20 degrees prior to the onset of fragmentation. So we use $\psi \approx 20$ degrees as a crude estimate, although the pre-shock ψ is likely to be somewhat larger than the pitch angle due to radial motion of the fluid as the wave becomes nonlinear. The fraction of mass f in the spirals just prior to fragmentation is also difficult to determine precisely but appears to be about 0.2. Using these values in (18) gives $\zeta \approx 16$. If we assume $\alpha_2 = 1$, then (17) becomes

$$\left(\frac{\delta r}{H}\right)^2 > \frac{11}{Q_i^3}, \tag{20}$$

Unfortunately, it is difficult to determine the true δr for the parts of the spirals that go into the dense fragments, and so it is difficult to verify that (20) is precisely satisfied. Nevertheless, for $Q = 1.25$ ($Q_i = 0.97$), (20) gives $\delta r \sim 3$ to $4H$ for the edge of the stabilized

region, or about six cell widths from the CR. The clumps are well within this distance from the CR in disk midplane images near the time of fragmentation.

Other aspects of our analysis can also be checked against the simulation results. For instance, the thinness of the arms in the midplane view of Fig. 1 suggests that condition (1) is satisfied, but this observation does not give us much insight into the proper value of α_1. Finally, consider m_f. Both Q_i and H in (15) vary linearly with c_i, while other parameters involved in Q_i and H, like κ, Ω, and Σ_1, are determined primarily by the disk's radial structure, which does not vary much with c_i for cool disks. So we expect m_f to be relatively insensitive to Q until the disk becomes hot enough for radial pressure gradients to play some dynamic role. In fact, our set of simulations with varied Q indicate that $m = 4$ is the fastest growing mode for all Q between 1.15 and 1.5 (Pickett & Durisen 2007b). The $Q = 1.0$ case has so many fast growing modes with various ms, including $m = 4$, that we cannot determine which of them grows most rapidly.

3.3. Fragmentation Criteria

Comparison of fragmentation criteria derived by different authors involves some discussion about how Q is evaluated. For our disk models, we use (11) but with the adiabatic sound speed (Pickett et al. 1998, 2000, 2003). One of the goals of our body of work has been to consider the same basic equilibrium disk models evolved under different assumptions about the equation of state of perturbed fluid elements. Because the models are derived from isentropic equilibrium configurations, it makes sense to use the adiabatic sound speed to compute a common reference Q that uniquely designates individual disk models within a set of related models. For the purposes of comparison with other work, however, we need to compute Q the same way other authors do, to the extent that we can. Johnson & Gammie (2003) and Mayer et al. (2004) use what we call here

Q_i. Johnson & Gammie (2003) report fragmentation if and only if $Q_i <$ about 1.4; the Mayer et al. (2004) paper suggests a somewhat larger value. We find fragmentation setting in for $Q < 1.7$ or, equivalently, for $Q_i <$ about 1.32, which is in reasonable agreement with these other works, especially considering that the precise limit in global simulations probably depends somewhat on the detailed structure of the disk.

Dynamically, it may seem obvious that the isothermal sound speed is the appropriate one to use in this case. However, the perturbations in our simulations are *locally* isothermal, i.e., the temperatures are fixed spatially throughout the grid. Sound waves traveling in the ϕ direction are truly isothermal, but waves moving in the r or z directions are not. Thus, it is not absolutely clear what sound speed should be used in our approximate stability relations, nor how truly "isothermal" the spiral shocks will be. This uncertainty is not explicitly accounted for by any of the α_js in our analysis. It also adds an unknown level of uncertainty when comparing gird-based locally isothermal simulations with SPH simulations in which the temperatures of the particles are kept fixed instead and thus may more accurately represent fluid elements that are evolving isothermally.

4. Conclusions

We have used simple anayltic arguments to estimate where fragments can first appear in a low-Q gravitationally unstable disk when the spiral shocks are locally isothermal. Assuming that the disk behavior is dominated by a single, coherent spiral mode and that the dense gas behind the spiral shock can be well approximated as a thin slab behind the shock, we have shown that compression from isothermal shocks will stabilize the slab against fragmentation away from the CR. The minimum distance from the CR at which the stabilization becomes effective gets larger for smaller Q_i. So the analysis suggests that, if fragments are going to form promptly upon nonlinear growth of a mode, then

this will happen first near the CR of the mode and only for low enough Q_i. The result is prompt formation of high density blobs near corotation in each spiral arm of the dominant pattern as it becomes nonlinear. A locally isothermal 3D hydrodynamic simulation of a well-studied star/disk model with $Q = 1.25$ ($Q_i = 0.97$) confirms this prediction. We plan to attempt similar analyses in the future to understand fragmentation under different conditions, including delayed fragmentation and fragmentation in radiatively cooled disks. We suspect that, as in the isothermal case, prompt fragmentation in radiatively cooled disks will tend to occur under more extreme conditions than fragmentation itself. In other words, if fragmentation generally occurs when $t_{cool}\Omega < c_{\gamma f}$ (Gammie 2001; Rice et al. 2003, 2005; Mejía et al. 2005), where $c_{\gamma f}$ is a constant that depends on the adiabatic index γ and where t_{cool} is the time it takes the disk to radiate away its internal energy, then we would expect prompt fragmentation to occur when $t_{cool}\Omega < c_{\gamma p}$, where $c_{\gamma p} < c_{\gamma f}$.

In closing, we would like to reiterate a crucial point we have made elsewhere (Pickett et al. 1998, 2000, 2003; Durisen et al. 2003; Mejía et al. 2005; Pickett & Durisen 2007a). The occurrence of fragmentation in disks does not in itself demonstrate that protoplanet formation by disk instability is really possible. The key issues are whether conditions for disk fragmentation are achieved in real disks (Rafikov 2005, 2006; Cai et al. 2006; Boley et al. 2006, 2007) and whether dense clumps, if they do form, are able to evolve into protoplanets over many orbital periods (Pickett & Durisen 2007a). Although our criteria suggest where potential precursors to gas giant planets *may* appear under certain restrictive conditions, they do not say anything about the longevity of the clumps once formed.

Acknowledgments

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Figure Captions

Figure 1. Midplane mass density grayscale. Shown is a grayscale intrepretation of the midplane mass density at the time when fragments first appear in the 3D hydrodynamics simulation of a $Q = 1.25$ disk given a cos4ϕ initial perturbation. The greyscale spans six orders of magnitude in density. The circle indicates the location of the corotation radius (CR). Note that the first clumps to appear are quite close to the CR for the four-armed mode.